PHASE NOISE MEASUREMENTS IN THE FREQUENCY DOMAIN

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ABSTRACT

This paper describes two measurement systems used to measure phase noise of sources in the frequency domain at Fourier frequencies from 10 hertz to 13 megahertz from the carrier. One system measures the combined phase noise characteristics of two sources. The two source signals are applied in quadrature to a phase sensitive detector (double balanced mixer) and the voltage fluctuations analog to the phase fluctuations are measured at the detector output. One measurement system is designed to measure the phase noise characteristics of a single oscillator. The single-oscillator measurement system is designed using delay lines to form an FM discriminator. Voltage fluctuations analog to the frequency fluctuations are measured at the detector output.

Basics of the Automated Program

A synthesizer serves as the local oscillator for a tracking spectrum analyzer. The Fourier frequency increments are chosen to be equal to the selected IF noise bandwidth of the spectrum analyzer. The range is from 2.5 seconds for the 3 hertz bandwidth, decreasing to 70 milliseconds for the 10 kilohertz bandwidth setting. Video smoothing is used in order to obtain a better approximation of the mean. The IF bandwidth settings for the Fourier frequency range selections are,

IF Bandwidth (Hz)	Fourier Frequency	IF Bandwidth (kHz)	Fourier Frequency (kHz)
3	10 - 400 Hz	1	40 - 100
10	400 Hz - 1 kHz	3	100 - 400
30	1 - 4 kHz	10	400 - 13 MHz
100	4 - 10 kHz		
200	10 - 40 kHz	(23 minutes running time)	

Amplitude auto-ranging is used to select the most sensitive range that does not result in overload.

General Theory and Definitions

The term frequency stability encompases any fluctuations of the output frequency of a device. Phase noise is the term most widely used to describe the characteristic randomness of frequency.

The Greek letter nu (ν) represents frequency for carrier-related measures. Modulation-related frequencies are designated (f). If the carrier is considered as dc, the frequencies measured with respect to the carrier are referred to as baseband, offset from the carrier, noise, modulation, or Fourier frequencies.

The spectral density $S_y(f)$ of the instantaneous frequency fluctuations $y(t)$ is defined as a measure of frequency stability.[6] $S_y(f)$ is the one-sided spectral density of frequency fluctuations on a "per hertz" basis.

$$S_y(f) = S_{\delta\nu}(f)/\nu_0^2 \qquad [\mathrm{Hz}^{-1}] \qquad (1)$$

$S_{\delta\nu}(f)$, in hertz squared per hertz, is the spectral density of frequency fluctuations calculated as $(\delta\nu_{rms})^2/B$. where B is the bandwidth used in measurement of $\delta\nu_{rms}$.

$S_{\delta\phi}(f)$, in radians squared per hertz, is the one-sided spectral density of the phase fluctuations on a "per hertz" basis.

$$S_{\delta\phi}(f) = (\nu_0^2/f^2)\, S_y(f) \qquad [\mathrm{rad}^2/\mathrm{Hz}] \qquad (2)$$

Script $\mathcal{L}(f)$ is defined as the ratio of the power in one sideband, referred to the input carrier frequency, on a per hertz of bandwidth spectral density basis, to the total signal power, at Fourier frequency (f) from the carrier per one device. It is a normalized frequency domain measure of phase fluctuation sidebands expressed in dB relative to the carrier per hertz [dBc/Hz].

$$\mathcal{L}(f) = \frac{\text{Power Density (one phase modulation sideband)}}{\text{Total Signal Power}} \qquad (3)$$

For the condition that the phase fluctuations occurring at rates (f) and faster are small compared to one radian, a good approximation is,

$$\mathcal{L}(f) = \frac{S_{\delta\phi}(f)\ \text{(one unit)}}{2\ \mathrm{rad}^2} \qquad [\mathrm{Hz}^{-1}] \qquad (4)$$

If the small angle condition is not met, Bessel function algebra must be used to relate $\mathcal{L}(f)$ to $S_{\delta\phi}(f)$.

$$\mathcal{L}(f)\ \text{in dB} = 10 \log \mathcal{L}(f) \qquad [\mathrm{dBc/Hz}] \qquad (5)$$

Phase Noise Measurement System Using Two Sources

Since 1967, NBS personnel have performed phase noise measurements using the type system shown in Figure 1. The double balanced mixer acts as a phase sensitive detector so that when two signals are identical in frequency and are in phase quadrature, the mixer output is approximately zero volts dc. The mixer output is a small fluctuating voltage δv centered on zero volts. The spectral density of phase fluctuations is given as,[2, 3, 8]

$$S_{\delta\phi}(f) = \frac{S_{\delta v}(f)}{K^2} = \frac{S_{\delta v}(f)}{2(V_{rms})^2} = \frac{1}{2}\frac{(\delta v_{rms})^2}{B(V_{rms})^2} \quad [\mathrm{rad}^2/\mathrm{Hz}] \quad (6)$$

where B is the noise power bandwidth of the spectrum analyzer used to measure the noise. K is the calibration factor in volts per radian. For sinusoidal beat signals, the peak voltage of the signal equals the slope of the zero crossing in volts per radian. Therefore, $V_{peak}^2 = 2(V_{rms})^2$.

The system of Figure 1 will now be described as it is used to obtain a direct measurement and plot of $\mathcal{L}(f)$ according to the following equation.

$$\mathcal{L}(f) = -\,[\text{Carrier power level} - (\text{Noise power level} - 6\ \mathrm{dB} + 2.5\ \mathrm{dB} - 10 \log B - 3\ \mathrm{dB})] \quad \text{in units of [dBc/Hz]} \qquad (7)$$

The measurement corrections are, - 6 dB which relates to the spectrum fold-over, the + 2.5 dB spectrum analyzer correction, the spectrum analyzer bandwidth correction, and - 3 dB since it is assumed that the two sources are similar and that the noise contribution is the same for each source.

Calibration and Measurement

1. Measure the noise power bandwidth of each IF bandwidth setting of the tracking spectrum analyzer.
2. Obtain a carrier power reference level (referenced to the output of the mixer).

3. Adjust for phase quadrature of the two signals applied to the mixer. Adjust the phase shifter to obtain an indication of zero volts dc at the mixer output.
4. Measure and plot the phase noise characteristics.
5. Measure and plot the system noise floor if desired.

The noise floor is obtained by repeating the automated measurements with attenuator No. 1 set to maximum or after disconnecting the unit under test and terminating the input of the mixer with a matched termination. The correction for noise floor is calculated as,

$$10 \log [(P_{\mathcal{L}(f)} - P_{noise\ floor})/P_{\mathcal{L}(f)}] \qquad (8)$$

Figure 2 is representative of the type of phase noise plot obtained using the system of Figure 1.

Phase Noise Measurement System Using a Delay Line to Form an FM Discriminator

The measurement system is shown in Figure 3. For maximum sensitivity in a power limited system the length of the delay line should have an attenuation value of one neper (8.686 dB) at the operating frequency.[1,7] The basic prodedure is as follows.

1. Measure the IF noise power bandwidths as required.
2. Establish the system power levels.
3. Adjust for phase quadrature (zero volts dc at the mixer output).
4. Calibrate the discriminator
5. Measure and plot the source phase noise.
6. Measure and plot the system noise floor.

Calibration of the discriminator usually requires replacing the unit under test with a modulateable source set to the same frequency and power level as the unit under test. At a selected Fourier frequency, modulation is applied to obtain the first Bessel null of the carrier as noted on a spectrum analyzer connected to coupler No. 1. The tracking spectrum analyzer is tuned to this modulation frequency and the power is measured. The power reading is corrected for the 50 dB setting of attenuator No.4 (which is required in order to perform the measurement without damaging the mixer in the spectrum analyzer). The correction is necessary since the attenuator will be set to zero dB during the measurement of the noise power.

$$P(dBm) = (- dBm \text{ power reading}) + 50 \text{ dB} \qquad (9)$$

$$V_{rms} = \sqrt{[(10^{P(dBm)/10})/1000] \times R} \qquad (10)$$

where R is 50 ohms in this system.

The discriminator calibration factor is calculated as,

$$CF = \frac{m \cdot f_m}{\sqrt{2}\, V_{rms}} = \frac{2.405\, f_m}{\sqrt{2}\, V_{rms}} \qquad [\text{Hz/volt}] \qquad (11)$$

The modulation index (m) is 2.405 for the first Bessel null as used in this technique. The modulation frequency is f_m. Other values of m could also be used.

Attenuator No. 4 is set to zero dB and the measurements are performed in the automatic mode. Noise power is measured at the selected Fourier frequency and is converted to the corresponding v_{1rms}.

$$v_{1rms} = \sqrt{[(10^{(P_n + 2.5)10})/1000] \times R} \qquad (12)$$

The rms frequency fluctuations are calculated as,

$$\delta\nu_{rms} = v_{1rms} \times CF \qquad (13)$$

The spectral density of frequency fluctuations is,

$$S_{\delta\nu}(f) = (\delta\nu_{rms})^2/B \qquad [\text{Hz}^2/\text{Hz}] \qquad (14)$$

The spectral density of phase fluctuations is,

$$S_{\delta\phi}(f) = S_{\delta\nu}(f)/f^2 \qquad [\text{rad}^2/\text{Hz}] \qquad (15)$$

$$\mathcal{L}(f) = 10 \log (S_{\delta\phi}(f)/2 \text{ rad}^2) \qquad [\text{dBc/Hz}] \qquad (16)$$

Corrections for the noise floor contribution can be calculated after measuring the noise floor with the delay line removed and the mixer and delay line terminated with matched loads. Increasing the power level into the mixer in order to obtain the same output impedance at which the source noise measurements were performed did not appear to affect the above measurement of noise floor.

Figure 4 illustrates the type of phase noise plot obtained using the single-oscillator measurement system of Figure 3. The plots were obtained using two different delay lines as noted. Beyond about 500 kHz the delay line discriminator is not calibrated and the plot is periodic in $\omega = 2\pi f_d$, where f_d is the Fourier frequency corresponding to $1/t_d$ and t_d is the delay of the delay line.[1]

The first null in the display occurs at 2 MHz on the plot for 600 MHz and the delay line length is therefore about 500 nanoseconds.

Comments

Chuck Reynolds of the Hewlett-Packard Company, Loveland Division, performed measurements in TRW Metrology using the program that he had developed for Hewlett-Packard Application Note 207. Measurements performed using the two-oscillator measurement system were in precise agreement with measurements performed using the TRW Metrology program.

Dr. J. Robert Ashley of the University of Colorado, Colorado Springs, and Gustaf J. Rast of the U.S. Army Missile Command, performed measurements of the phase noise characteristics of TRW developed Surface Acoustic Wave Oscillators with their delay line FM discriminator system at Redstone Arsenal in Huntsville, Alabama. The measurements were in close agreement with measurements performed with three different systems in TRW Metrology.

Joe Oliverio of Hewlett-Packard performed phase noise measurements in TRW Metrology using the Hewlett-Packard 5420 Digital Signal Analyzer which measures from sub-millihertz to 25 killohertz from the carrier. Measurements were in precise agreement with measurements performed using the TRW Metrology program.

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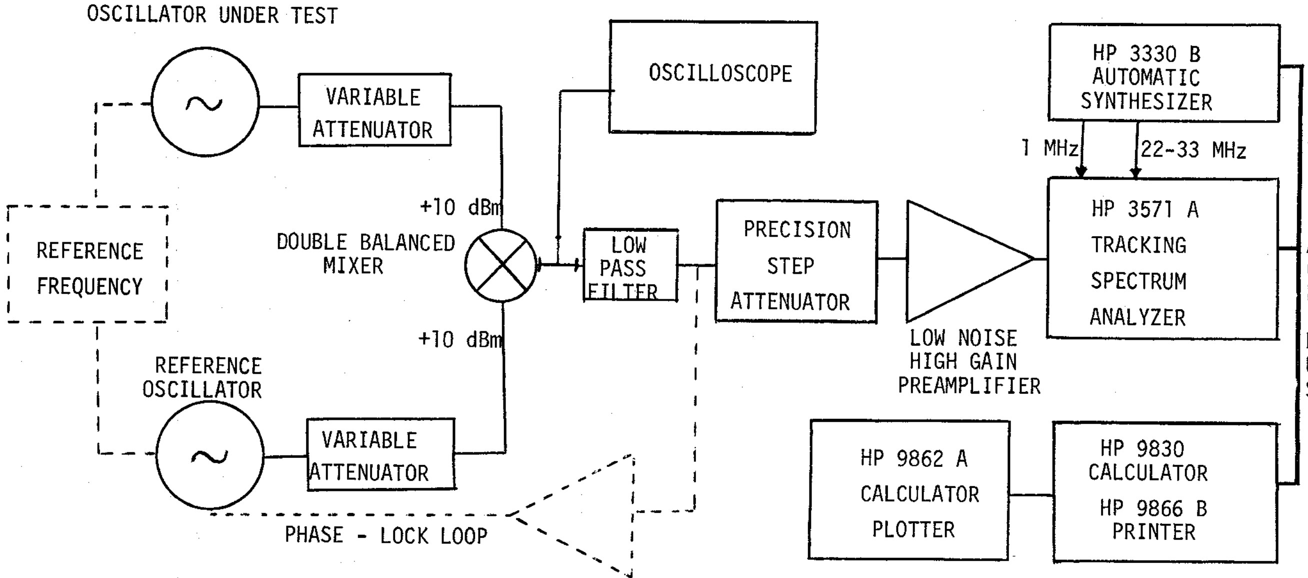

FIGURE 1. PHASE NOISE MEASUREMENT SYSTEM USING THE TWO - OSCILLATOR TECHNIQUE.

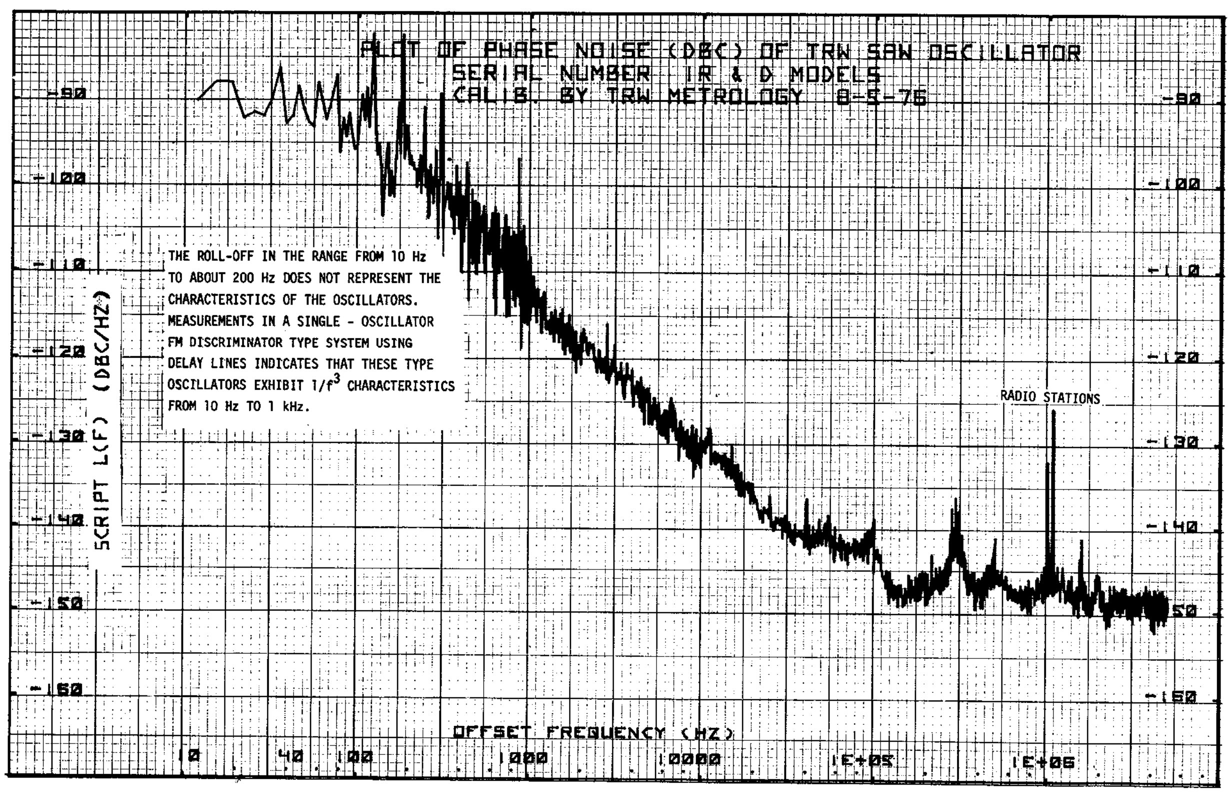

FIGURE 2. PHASE NOISE PLOT USING THE TWO-OSCILLATOR TECHNIQUE.

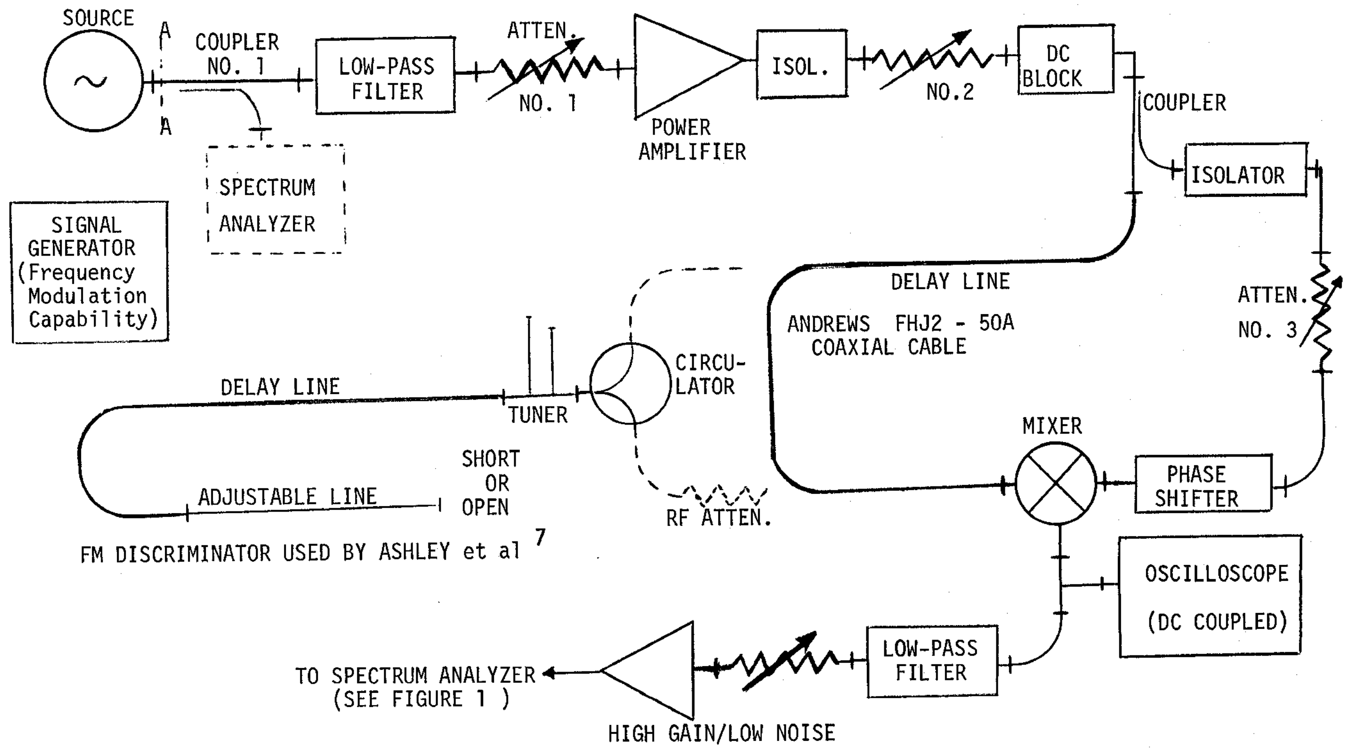

FIGURE 3. SINGLE-OSCILLATOR PHASE NOISE MEASUREMENT SYSTEM USING THE DELAY LINE AS AN FM DISCRIMINATOR.

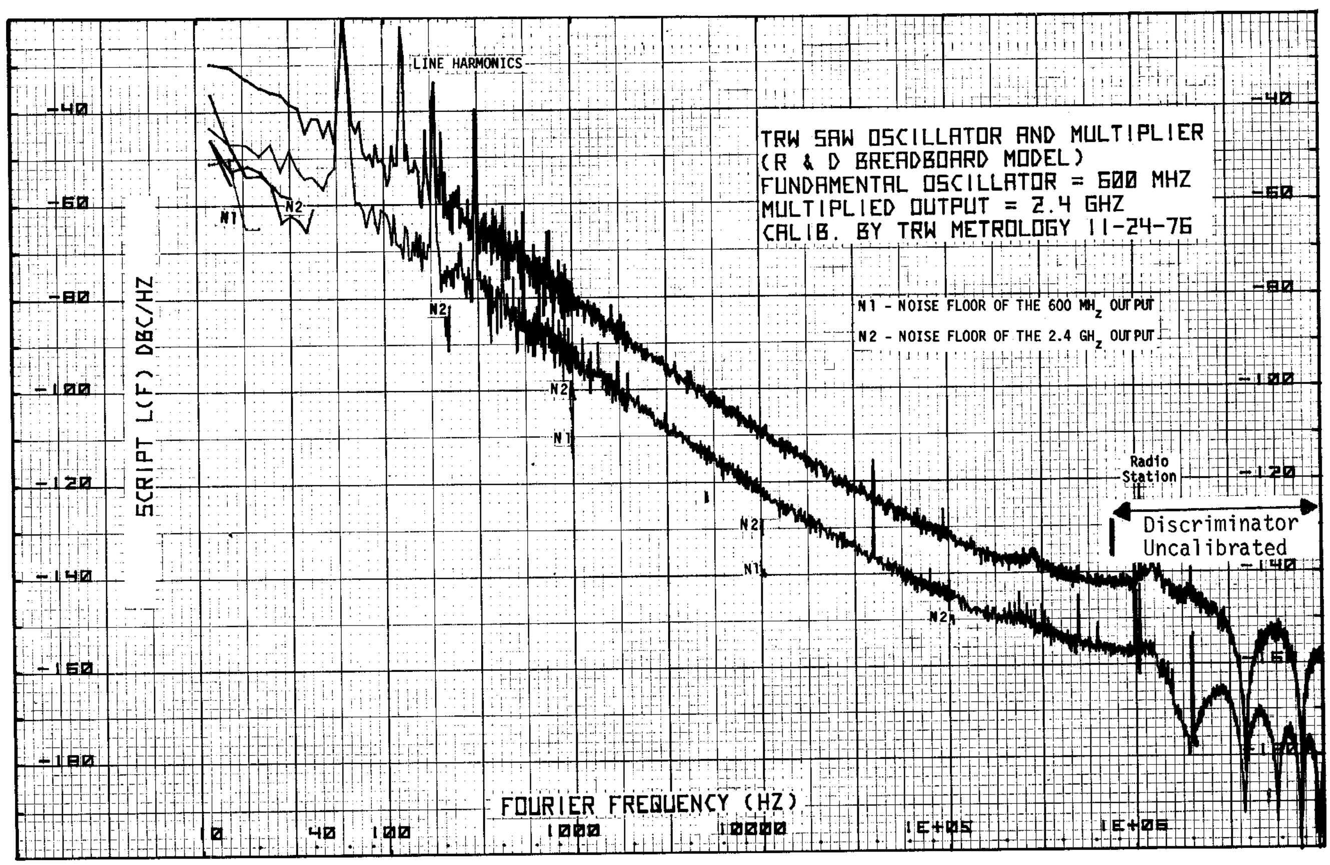

FIGURE 4. PHASE NOISE OF 600 MHZ OSCILLATOR MULTIPLIED TO 2.4 GHZ